

**FACULTY OF SCIENCE
UNIVERSITY OF THE WITWATERSRAND
JOHANNESBURG**



**Investigating Challenges that Grade 11 Mathematics Learners
Face when Translating from Word Problems to Linear Algebraic
Representations**

By

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Declaration

I declare that this research report is my own, unaided work. It is being submitted for the Degree of Master of Science in the University of Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.



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Signature.

Date..... December 2014

Abstract

The National Curriculum and Assessment Policy Statement document (CAPS) (Department of Basic Education, 2012: 4) outlines some of the salient skills related to the modelling of word problems that Mathematics learners are expected to acquire during the course of their learning. One such critical skill is the learners' ability to represent and describe situations in algebraic language, formulae and expressions. In line with this broad objective, the present study investigated learners' linguistic knowledge and skills in translating from word to linear algebraic representations. Particularly focussing on errors learners made during the translation process, a class of 40 Grade 11 learners at one school in Gauteng West District were selected and given a written test on word problems. To gain access to learner misunderstandings leading to the observed errors and their sources, five learners were selected for interviews that were audio recorded. A Mixed-method Sequential Explanatory Design was used in the study. The major finding of this present study was that learners committed more semantic errors than syntactic and schematic errors suggesting that they (learners) had challenges in understanding the language used in the word problems. The major sources of semantic errors were found to be learners' lack of vocabulary knowledge, inexperience in interacting with expository text structures and their lack of syntax awareness. In addition to the above finding, learners exhibited limited metacognitive skills essential in word problem solving as well as their inability to reflect on the appropriateness of their written algebraic representations for all categories of word problems. The study concludes by providing recommendations on how errors committed in word problem solving can be attended to.

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